Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Numerical Methods

Subject Code: 4SC04NUM1 **Branch: B.Sc. (Mathematics, Physics)**

Semester: 4 **Date:24/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) State Weddle's rule. (02)

b) Using Euler's method, the value of $y(0.1)$ for $y' = 1 - \frac{2x}{y}, y(0) = 1$ is _________. (02)

c) In the second derivative using Newton's forward formula, what is the coefficient of $\Delta^2 f(x_0)$ is (01)

1) $-\frac{1}{h}$ 2) $\frac{1}{h^2}$
3) $-\frac{1}{h^2}$ 4) $\frac{1}{h}$

d) The root of the equation $f(x) = 0$ in the interval (a, b) is given by (01)

1) $\frac{af(b) - bf(a)}{f(b) - f(a)}$ 2) $\frac{bf(a) - af(b)}{f(b) - f(a)}$
3) $\frac{bf(a) - af(b)}{b - a}$ 4) $\frac{af(b) - bf(a)}{b - a}$

e) Varies type of Runge-Kutta methods are classified according to their (01)

1) degree 2) rank
3) order 4) none of these

f) Taking $n = 4$, trapezoidal rule gives the value of $\int_1^2 \frac{dx}{x}$ is (01)

1) 0.679 2) 0.673
3) 0.637 4) 0.697

g) The method which do not require the calculations of higher order derivatives is (01)

1) Taylor's method 2) R-K method
3) both 1) and 2) 4) none of these

h) Milne's corrector formula is (01)

1) $y_4 = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4)$ 2) $y_4 = y_2 - \frac{h}{3}(f_2 + f_3 + 4f_4)$
3) $y_4 = y_2 + \frac{h}{3}(f_2 + f_3 + 4f_4)$ 4) $y_4 = y_2 - \frac{h}{3}(f_2 + 4f_3 + f_4)$

	i)	The Newton-Raphson iterative formula for finding $\frac{1}{N}$ is $x_{i+1} = x_i(2 - Nx_i)$. (True/False)	(01)
	j)	Taylor's series method will be useful to give some starting values of Milne's method. (True/False)	(01)
	k)	The second order Runge-Kutta formula is Euler's method. (True/False)	(01)
	l)	Newton-Raphson method is applicable to the solution of both algebraic and transcendental equations. (True/False)	(01)

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Find the roots of the equation $e^x - 3x = 0$ correct up to two decimal places with lies between 1 and 2 by using bisection method.	(05)
	b)	By the method of iteration, find the root of the equation $x^2 - \sin x = 0$ correct up to four decimal places with lies between 0.5 and 1.	(05)
	c)	Show that Newton-Raphson method has second order convergence.	(04)
Q-3		**Attempt all questions**	**(14)**
	a)	Compute a root of $x\, ln\, x - 1 = 0$ correct to three decimal places by Regula-Falsi method.	(05)
	b)	Derive Newton's iterative formula for finding q^{th} root of a given number N and hence find the value of $\sqrt[5]{3}$ correct up to four decimal places.	(05)
	c)	Evaluate $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\ dx$, by Simpson's three-eighth rule, taking $h = 0.2$, correct to five decimal places.	(04)
Q-4		**Attempt all questions**	**(14)**
	a)	Derive Trapezoidal rule.	(10)
	b)	Find the positive root of $x^3 + x - 1 = 0$ correct up to five decimal places by using Newton-Raphson method.	(04)
Q-5		**Attempt all questions**	**(14)**
	a)	Evaluate $\int_{0.1}^{0.7} (e^x + 2x)\, dx$ by using Trapezoidal and Simpson's one-third rule, taking $h = 1$, correct to four decimal places.	(06)
	b)	Describe Picard's Method for first order ordinary differential equation.	(04)
	c)	Using Taylor series method, find the values of $y(0.1)$ and $y(0.2)$, given $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$, correct up to five decimal places.	(04)
Q-6		**Attempt all questions**	**(14)**
	a)	Compute $y(0.6)$, by Runge-Kutta method correct to five decimal places, from the equation $\frac{dy}{dx} = xy,\ \ y(0) = 2$, taking $h = 0.2$.	(05)
	b)	Find $y(0.1)$, by Euler's method, from the differential equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ when $y(0) = 1$, correct to four decimal places, taking step length $h = 0.02$.	(05)
	c)	Obtain Picard's second approximate solution of the initial value problem $\frac{dy}{dx} = \frac{x^2}{y^2+1}, y(0) = 0$.	(04)
Q-7		**Attempt all questions**	**(14)**
	a)	Compute $f'(3)$ and $f''(3)$ from the following table	(06)

x	0	5	10	15	20
$f(x)$	1.5708	1.5738	1.5828	1.5981	1.6200

	b)	Derive differentiation formulae based on Newton's Backward formula.	(05)

	c)	Apply Euler-Maclaurin sum formula to find the sum $1^3 + 2^3 + 3^3 + \cdots + n^3$.	(03)
Q-8		**Attempt all questions**	**(14)**
	a)	Compute $y(0.5)$, by Milne's predictor corrector method from $\frac{dy}{dx} = 2e^x - y$ given that $y(0.1) = 2.0100, y(0.2) = 2.0401, y(0.3) = 2.0907, y(0.4) = 2.1621$.	(05)
	b)	Given $\frac{dy}{dx} = 1 - \frac{y}{x}$ when $y(2) = 2$, compute $y(2.1)$, by Euler's modified method, correct up to four decimal places, taking $h = 0.05$.	(05)
	c)	Derive differentiation formulae based on Newton's divided difference formula.	(04)